

Hypercontractivity

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Abstract. In this mini course we will give an introduction to hypercontractivity both in the classical and noncommutative settings. We will also discuss its links with Sobolev and logarithmic Sobolev inequalities. Below is the plan of the course.

- **Lecture 1:** Two-point inequality and hypercontractivity for the Walsh system (or equivalently the dyadic group); Khintchine, Khintchine-Kahane and Khintchine-Borel inequalities.
- **Lecture 2:** Gaussian hypercontractivity, i.e., hypercontractivity for the Ornstein-Uhlenbeck semigroup; more classical examples.
- **Lecture 3:** Links with log-Sobolev and Sobolev inequalities.
- **Lecture 4:** Noncommutative case, hypercontractivity for anticommutation relation (Fermi field).
- **Lecture 5:** Hypercontractivity for free Gaussian variables (semicircular systems) and for free groups; open problems.